A SPURRED SPIRAL RAMP FOR THE GREAT PYRAMID OF GIZA

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Abstract

An easier and equally feasible configuration of spiral ramps for the construction of the Great Pyramid of Giza (Brichieri-Colombi, 2015), would be for a spiral ramp extended as a spur tangential to the pyramid rather than orthogonal to it. The general arrangement, which could have been used for many other large pyramids as well, is similar to that proposed by Lehner (1985: 129-132), but without the mass of temporary works that Lehner envisaged. It avoids the need to create a trench over the body of the pyramid during construction, as proposed by Arnold (1991: 98), while respecting the constraints imposed by the available tools, workforce capabilities and design features of the pyramid. Finding the ideal configuration would not have been easy for the ancient builders, but this paper demonstrates how they could have done so with models. It also addresses the key construction issues associated with spiral ramps. An analysis of the construction effort required demonstrates that a ramp slope of 1:6 (9.5⁰) would have minimised the work involved. This finding suggests that pyramid construction hypotheses should be evaluated in terms of both feasibility and optimality to assess which are the most likely to have been adopted by ancient Egyptians.

Abbreviations

H - hour; km - kilometres; m - metres; M - million; s - second; t - tonne; t-m - tonne-metres torque

Note that the expression "at level n" makes reference to a point or surface at a level of n metres above the foundation level of the Great Pyramid.

Introduction: Feasible Ramp Hypothesis

While building a two-metre high sculpture (figure 1) based on the hypothesis for a feasible ramp configuration for the Great Pyramid founded on a combination of straight and spiral ramps (Brichieri-Colombi, 2015), it became apparent that an alternative configuration, which avoided the complication of a trench within the body of the pyramid, would have been easier. Accordingly, both the hypothesis and the sculpture have been modified (figure 2).

The trench, as proposed by Arnold (1991: Fig 3.53 (4)), was designed to allow the heaviest beams to be hauled up to their destination levels by large teams on a straight ramp. As shown below in section 'Intersections of Ramp and Pyramid' this objective can be achieved more easily with a tangential ramp that avoids the need for a trench.

Key Parameters and Changes in Values

For convenience, the values of the key parameters that are used in formulating the new hypothesis are shown in table 1, together with the principle source. Changes from assumptions made in Brichieri-Colombi (2015) are listed below:

1. The slope of the ramp is assumed to be 1:6 (a *seked* of 6). This corresponds to 9.462°, close to the value of 9.314° adopted previously (Brichieri-

Figure 1. Spur ramp sculpture.

Figure 2. The original (left) and revised (right) configurations.

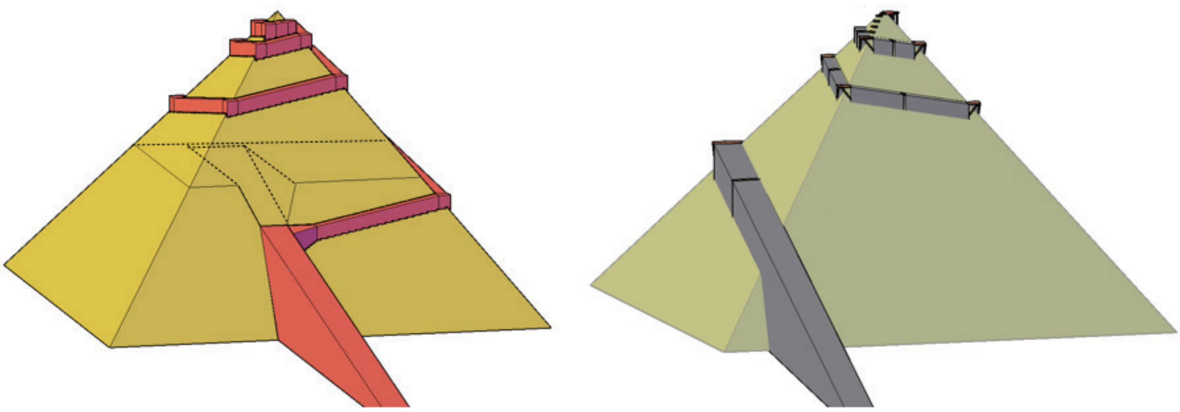


Table 1. Key parameters.

	Value	Unit	Source & Notes
Pyramid			
Elevation of foundation (above sea level)	60.418	m	Cole (1925)
Width of base at top of socle	230.329	m	Dash (2012)
Core footprint side	228.592	m	Petrie (1883)
Slope of apothem 51.843⁰	1.273	ratio	Cole (1925)
Slope of arris 41.986⁰	0.900	ratio	Calculated
Height of finished pyramid	146.573	m	Calculated
Length of arris	162.867	m	Calculated
Height of core	145.468	m	Calculated
Excess width of casing blocks	0.050	m	Assumed
Heaviest granite beam above King's Chamber	75	t	Brichieri-Colombi (2015)
Weight of pyramidion (as Red Pyramid)	2.4	t	Dorner (1998)
Eqn. defining heaviest load passing level h	$108\ h^{-0.788}$	t	This paper
Friction under sled runners μs	0.25	ratio	Measured
Friction between foot and ramp μf	0.65	ratio	Li & Wen (2013)
Biometric			
Average weight of haulers	67.9	kg	Brichieri-Colombi (2015)
Max individual haul force parallel to ramp	43.5	kg	Brichieri-Colombi (2015)
Allowance for team effect hd	0.78		Kravitz & Martin (1986)
Men per tonne of load hauling up ramp	12.0	no.	Calculated
Men per tonne of load hauling down ramp	2.4	no.	Calculated
Weight of men to haul load	82	%	Calculated
Number of men for largest load	896	no.	Calculated
Maximum short-term overload *e.g.* at corners	30	%	Assumed
Maximum angle turned through at corner	60	°	Calculated
Minimum lateral spacing of haulers	0.80	m	Brichieri-Colombi (2015)
Minimum longitudinal spacing of haulers	0.65	m	Brichieri-Colombi (2015)
Men abreast in haul team for teams < 100 men	4	no.	Assumed
Men abreast in haul team for teams > 100 men	8	no.	Assumed
Safety margin around team (front, rear & side)	1.0	m	Brichieri-Colombi (2015)
Maximum men per haul rope (to avoid breaking)	73	no.	Calculated
Sustainable power output per hauler	100	W	Brichieri-Colombi (2015)
Speed on ramp (2.1 km/h)	0.570	m/s	Calculated
Walk-back speed (3.6 km/h)	1.000	m/s	Assumed
Loading time	10	min	Assumed
Contingency on time	10	%	Assumed
Working day	8	hrs	Brichieri-Colombi (2015)
Working days per year	300	days	Brichieri-Colombi (2015)

Colombi, 2015: 7), but in all the formulae it is expressed as a variable. Later in this paper, tests indicate the likelihood that this assumption is correct.

2. A factor is introduced to allow for the so-called Ringelmann effect (Kravitz & Martin, 1986). This factor reflects the fact than men hauling in teams become less efficient as team numbers increase, the factor reducing asymptotically to 75%. This increases the haul team length as the number of men required to haul a loaded sled up a ramp with a 1:6 slope increases from 9.4 to 12.0 men per tonne. Ayrinac (2016: 468: 9) refers to the importance of this effect but ignores it in his calculations on the assumption that team efforts could be synchronized by a man giving a rhythm, indicating that he has misunderstood Ringelmann's analysis. Analyses by de Haan (2009) and Monnier (2020) make no mention of this effect.

3. Because of the longer team length, the maximum level that is served by the ramp decreases from 142 m (Brichieri-Colombi, 2015: 13) to 136 m, as the flight of the ramp becomes too short. The lifting mechanisms that are proposed for the capstones would thus have had to be introduced at a lower level to raise the core and casing blocks in the uppermost courses as well. As demonstrated below, the mechanism that is proposed previously (Brichieri-Colombi, 2015: 13) works equally well at these lower levels.

4. The width of the ramp on the spur has been reduced slightly from 14.0 m (Brichieri-Colombi, 2015: 8) to 11.6 m to correspond to the calculated minimum width.

5. The estimated weight of the pyramidion for the Great Pyramid has been revised upwards from 1.5 t (Brichieri-Colombi, 2015: 5) to 2.4 t. The Dashur pyramidion on display near the Red Pyramid was found in many pieces in 1982 by Rainer Stadelmann. It is either from the Red Pyramid itself, or, as Corinna Rossi (1999: 222) has suggested, from the Bent Pyramid, and discarded when the angle was changed. The pieces were measured by Josef Dorner, who also worked on the pyramidion of the satellite pyramid at Giza (Stadelmann, 2009: 312). The pyramidion was roughly assembled without plaster, later reassembled with plaster, and after being vandalized, it was cleaned and reassembled in 2006 in accordance with Dorner's measurements. The reconstruction is 1.00 m high and the base width 1.57 m, and therefore has the same slope as the Great Pyramid. If carved from Tura limestone, its weight would have been 2.4 t.

The pyramidion of the 30 m high Queen's satellite pyramid (Hawass, 1995: 105-124) was of Tura limestone and from the measurements by Dorner (1998: 105-124) its weight can be calculated as 0.90 t. Of the four pyramidions in the Cairo Museum, the largest is the basalt pyramidion of the 53 m high pyramid of the 12th Dynasty pharaoh Amenenhat III, with a weight of almost 5 t (Maspero, 1903: 154). Clearly, there is no relationship between the height of the pyramid and the size of the pyramidion.

The pyramidion at Dahshur is the closest example in time and space of a pyramidion for a major pyramid, and in this construction analysis it is therefore assumed that the pyramidion of the Great Pyramid was at least as heavy.

Spurred Ramp

The first premise of the straight-spiral ramp is that the heavy beams above the King's Chamber had to be hauled to their final height on a straight ramp, because the long teams that would have been required would not have been able to haul around a curve, kink or corner. Table 2 shows the estimated weight and level of the heaviest beams in each layer of the structure, and the length of team required to haul each.

Chamber roof	Density	Elev. of base	Length	Depth	Width	Wt.	Team size	Length to sled CL	Min. level at corner
	t/m³	m	m	m	m	t	No	m	m
King's	2.64	48.84	8.50	2.15	1.55	75	934	82	62.3
Davidson's	2.64	51.84	7.70	2.00	1.40	57	717	64	62.4
Wellington's	2.64	54.54	7.30	2.00	1.30	50	625	57	63.9
Nelson's	2.64	57.44	7.30	2.25	1.45	63	792	70	69.0
Lady Arbuthnot's	2.64	60.14	7.00	2.05	1.20	45	567	52	68.7
Campbell's	2.17	62.27	11.22	1.80	1.25	55	684	62	72.4
Minimum									72.4

Table 2. Target level for top of main ram.

Note:

1) All roofs of granite, except Campbell's, of limestone.

2) Length to sled centre line is team length for eight men abreast plus safety margin front and rear, plus half sled length. This assumes all men stood on the slope, with top man at the north west pyramid corner.

The ramp would have had to be high enough to ensure that, when the front man of the team was at the top to the ramp, the sled was at least at the level of its intended position. When the analysis in table 2 is repeated over a wide range of slopes, it is always the limestone beams of the uppermost gable roof that are the ones that determine the minimum level that the ramp had to reach. For a 6-*seked* ramp, this level is 74 m, and varies only little with slope.

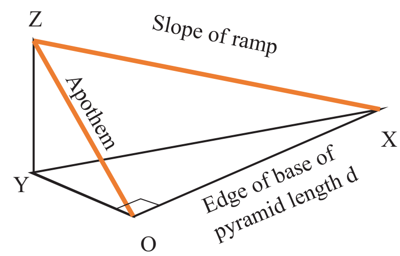
The second premise is that all construction materials, except the limestone blocks extracted from the immediate vicinity, originated either from the quarries on the southern part of the Giza plateau, or from elsewhere, arriving at the site via a harbour near the sphinx (Lehner, 2009: 46). It had to be brought up a 10 m wide natural bench of rock which was dressed to become what is now called Khafre's causeway. Thus, all materials had to cross the northern edge of this causeway, whose location, elevation and orientation is known accurately from the Egyptian Survey Authority 1:5000 contour map of the plateau.

Role of Drawings, Models and Calculations in Design

The design of a spiral ramp and the location of its starting point to arrive at a predetermined level on the pyramid would have required architectural drawings and calculations, and/or models. Arnold (1991: 7-11) reviews the possible use of drawings, and concludes that, although ancient Egyptians were certainly capable of preparing them, "no true building plan as executed by an architect for construction purposes has been preserved". He shows an illustration of a 3rd Dynasty ostracon, found at Saqqara, which shows a sketch for the construction of a vaulted roof, but nothing from then until the 11th Dynasty. Similarly, although they were capable model builders, Arnold (1991: 9) shows there is no evidence of models being used for architectural purposes, at least until the 12th Dynasty.

The Moscow Papyrus dating to the 12th Dynasty, shows the ability to do calculations and solve certain problems (Struve & Turaev, 1930). It includes a formula for the volume of a frustum (Problem 14) that only could have been derived had they known how to calculate the volume of a pyramid. This volume can only be calculated by a process akin to integration, or the summing of a series, but there is no indication that the Egyptians were aware of either process until Graeco-Roman times (Clarke & Engelbach, 1929: 223). However, it would have been easy for the Egyptians to demonstrate empirically that the volume of a pyramid is equal to ⅓ base area times height by making a model of a pyramid and a container with the same height and the same base. They could then have observed that, if they filled the inverted pyramid with dry sand and tipped it into the container three times, the container would be filled to the brim with no excess, whatever the shape of the base. This supports the idea that models might well have been used to express mathematical concepts, especially those involving non-rectangular solids which would have been difficult to draw. Many of the ar-

The diagram shows a ramp 1 unit long with slope r whose top edge lies against the side of a pyramid with an apothem slope of a.



The slope s of the ramp, when viewed in elevation from the side of the pyramid, is (r/d)

From Pythagoras, $d = \sqrt{(1-(r/a)^2)}$. Hence

$s = r/\sqrt{(1-(r/a)^2)}$

The tangent of the angle between the base of the ramp and the side of the pyramid, expressed as a slope p, is (r/a)/d. Since s = r/d, this can be expressed as

$p = s/a$.

Frame 1. Ramp angles in elevation and plan.

guments that follow are based on calculation but could also have been made with the simple models that are shown.

Intersections of Ramp and Pyramid

We can calculate the orientation of the line of intersection of the plane of the ramp and the face of the pyramid from their respective slopes as 7⁰ in plan view (relative to the line of the side) and 10⁰ in elevation (frame 1). This calculation requires the ability to extract square roots and express angles in plan view, and it is far from certain that the ancient Egyptians could do either of these operations 800 years before the Moscow papyrus was written. However, they could have determined these angles easily enough (figure 3), using a pyramidion similar to the one at Dahshur (which has the same apothem slope as the Great Pyramid) and a cut-out of the ramp with a 1:6 slope. All that would have been required was to place the bottom end of the cut-out against a corner of the pyramidion, rotate it while keeping it vertical until it lay against the face of the pyramidion, and trace the line where the cut-out met the face.

It would have been desirable to have a smooth transition for the haul teams between the causeway and the bottom of the ramp, which, as calculated above, would have to be oriented at 7⁰ to either the west or east edge of the pyramid. We

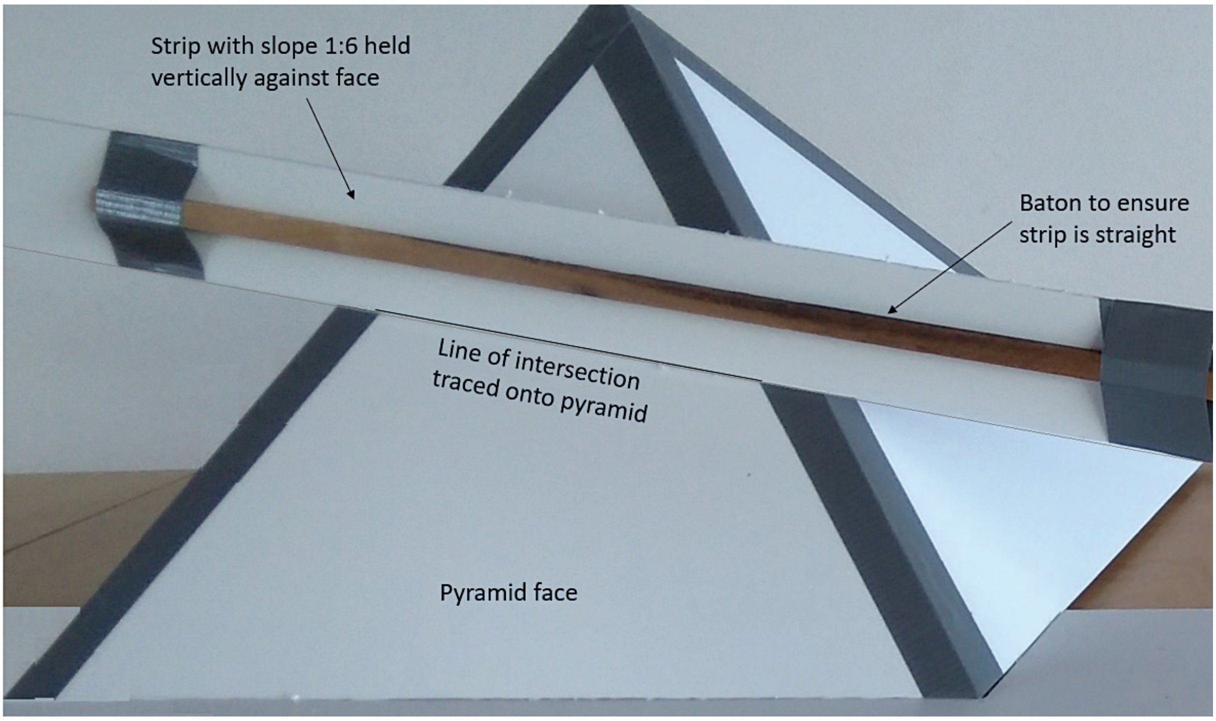
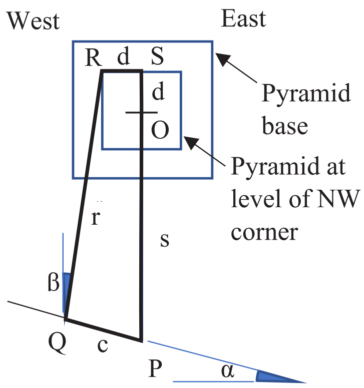


Figure 3. Measurement of line of intersection of ramp and pyramid.

Variables
p = Height of pyramid
u = Slope of apothem
w =Slope of ramp
v = Slope of causeway
α = Angle of causeway north of west
β = Angle of ramp to edge of pyramid base



Unknowns
h = Level of NW platform
d = (Width of pyramid at level h)/2
c = Distance PQ in plan view
r = Distance RQ in plan view

Solve 4 equations in 4 unknowns (h, d, c, r) for ramp on west side
1) EW direction: $d - c.\cos(\alpha) + r.\sin(\beta) = 0$
2) NS direction: $-d + c.\sin(\alpha) + r.\cos(\beta) = s$
3) Vertical direction: $h - c.v - r.w = -t$
4) Pyramid shape: $h + u.d = p$
Note: For ramp on east side, sign of c in equation (2) changes
2) NS direction: $-d - c.\sin(\alpha) + r.\cos(\beta) = s$

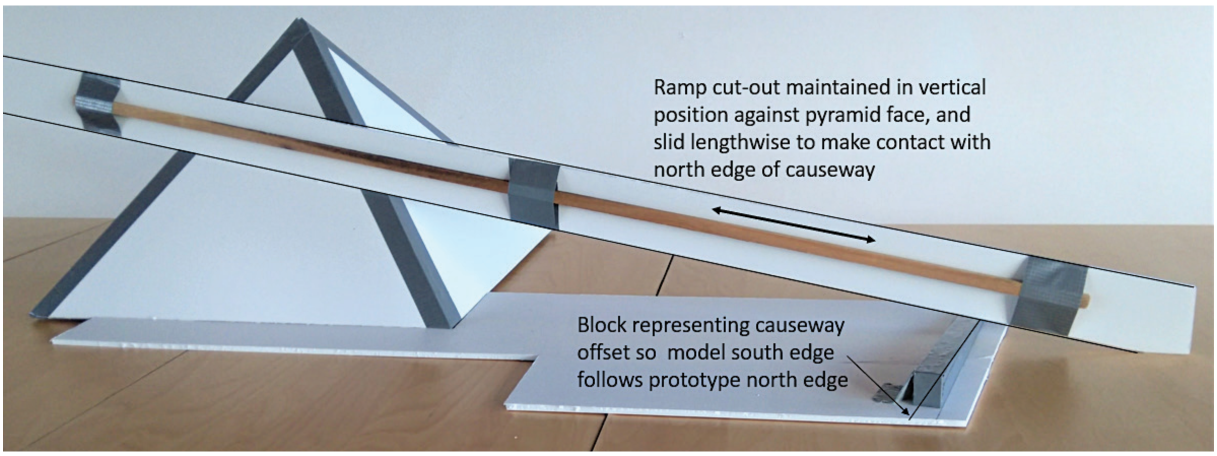
Frame 2. Intersection of ramp with pyramid and causeway.

can calculate the location of the intersection of the lines marking the inner edge of the ramp and the northern edge of the causeway by solving the three simultaneous linear equations that describe the location of the causeway relative to the centre of the base of the pyramid in each dimension, and a fourth that represents the shape of the pyramid. The solution tells us where along the causeway the intersection point lies, and the level at which the ramp touches the north-west, or north-east, arris of the pyramid (frame 2).

A review of Egyptian mathematics (Katz & Imhausen, 2007: 9-44) shows that it was largely procedural and, although it is clear that ancient Egyptians were experts at manipulating ratios, there is no evidence that they could solve sets of simultaneous equations. However, they could have determined this point exactly by modelling, using the same pyramidion and cut-out as before, and a wedge of limestone with the appropriate slope to represent the causeway. The distance from the south west corner of the pyramid to the causeway was 241 m, so if the model scale (the ratio of the height of the pyramidion to that of the pyramid) had been 1/100, the wedge could be located 2.41 m from the corner, and oriented at the same angle to the pyramidion.

By keeping the cut-out against the edge of the pyramid while straddling the causeway, they could have moved it lengthwise to the position where it just

Figure 4. Measurement of location of intersection of ramp and causeway.



touched the edge of the causeway (figure 4). In the model in figure 4, at a scale of 1:500, the difference between the model and calculated dimensions corresponded to an error in the prototype of less than 0.5 metres. A larger scale model would have been even more accurate.

For a 6-*seked* ramp on the west, the start location would have been approximately due south of the south-west corner of the pyramid, and the top level 75 m, just above the required minimum level. For a ramp on the east, the start location would have been approximately due south of the south east corner, and the top level 84 m.

The concept of scale models was clearly well understood, as evidenced by the range of scales used in the production of statues. We would not expect them to be preserved once the prototype had been built, as they were devoid of interest compared with statues.

Although the eastern ramp would have reached a higher level, and thus appear more attractive, the spur would have crossed over the quarry between the causeway and the pyramid. If, as is possible, this quarry was opened later for Khafre's pyramid, the eastern spur would have been feasible, and thus its presence cannot at this stage be eliminated. Both ramps could have been constructed if the rate of placing had been an issue, but a single ramp with three tracks (two working and one for overtaking) would have been adequate to build the pyramid within a 23-year timespan (Brichieri-Colombi, 2015: 8). Once the start location and the slope of the ramp had been determined, the elevations of each corner platform could also have been determined, either mathematically (frame 3), or with a template triangle drawn on the west side, bounded by the line of the ramp, and the north and south arrises. The template could then have been used to trace the line of the inner edge of the ramp around successive sides of the pyramid (figure 5).

The maximum level it could have reached was constrained by one of two levels: the level at which the vertical interval between successive circuits was less than the height of the vertical outer wall of the ramp; or the level at which the length of the flight of the ramp against the pyramid, plus the ramp width, is shorter than the length of the team required to haul the maximum load to be hauled up the flight. These values can be calculated (frame 4) or measured directly on a model of the pyramid.

Highest Possible Ramp Level

Using the maximum loads calculated at each level (Brichieri-Colombi, 2015: 5), it transpires that the length of the ramp is the controlling factor, and sets an upper limit of 136 m. Above this level, which would have been reached at the third south west corner platform, alternative lifting arrangements would have been

The diagram shows a flight of the ramp viewed from the side of the pyramid. The slope of the pyramid apothem is a and the slope of the ramp in elevation is s, as calculated earlier. The vertical elevations at the start and end of the flight are g_1 and g_2 and the horizontal distance between them is d. Since

$d = g_1/a + g_2/a$

and

$g_1 = g_2 + s.d$

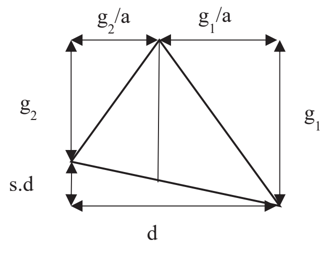
we can eliminate d to get

$g_2 = k. g_1$

where

$k = (a - s)/(a + s)$

Since a and s are constant, k is constant and g_2 is always a fixed proportion of g_1



Frame 3. Calculating corner levels.

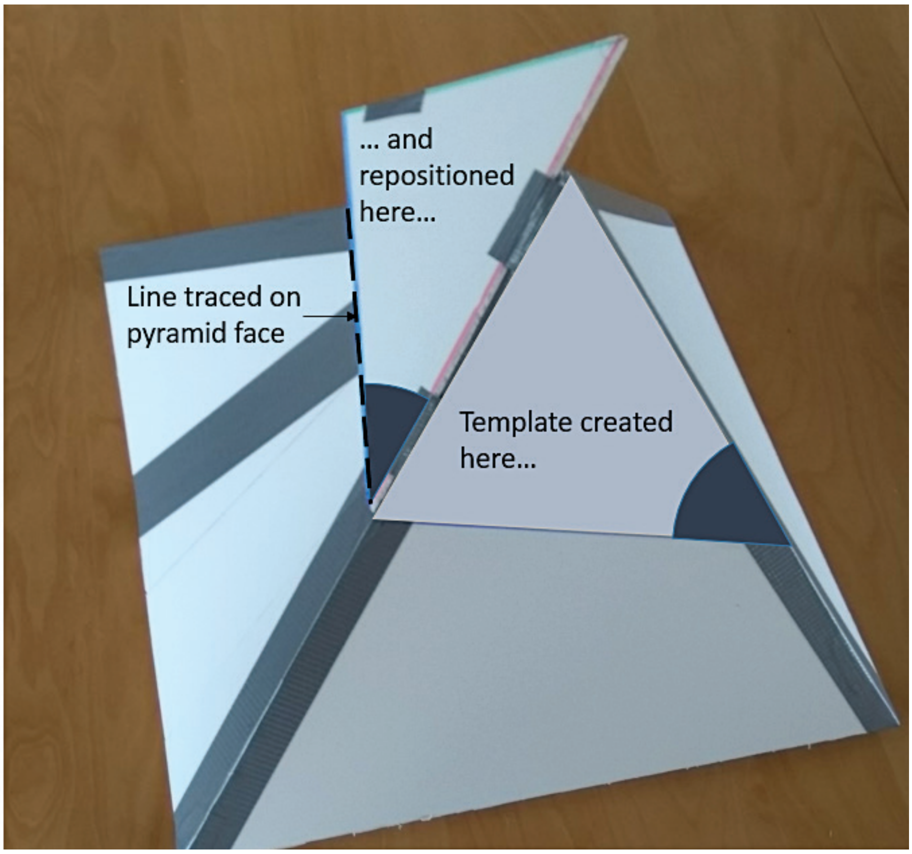
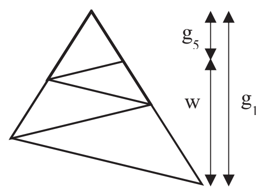


Figure 5. Template to mark corner levels.

1) As governed by vertical interval

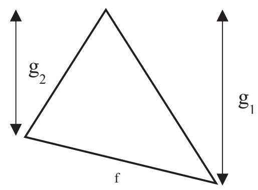


The flights of the ramps become closer and closer as they rise. At the limit, the vertical interval between successive flights on the same face is equal to the height w of the outer wall of the ramp, which is a function of its width.

As shown above, on a single face, $g_2 = k.g_1$, so after a complete circuit of the pyramid, the top level g_5 is $k^4 g_1$ and the vertical difference w is $g_1.(1 - k^4)$. Hence

$$g_5 = w.k^4/(1+k^4)$$

2) As governed by flight length



The flights of the ramps also become shorter and shorter as they rise. From the three-dimension version of Pythagoras' theorem, the length f of each flight is

$$f = (1/a)\ \sqrt{(2*(g_1 - g_2)^2 + (g_1 + g_2)^2\ )}$$

The length available for the haul team will be this length, plus the width of the ramp at the corner platform at the upper end.

Frame 4. Highest possible ramp level.

required. The ramp would nevertheless have been needed to provide ease of access to higher levels and for the removal of trimmings from the facing blocks when the faces were smoothed to the glacis slope. For downslope hauls, teams would have been only 20% as large as those for upslope hauls, so the width of the ramp could have been reduced to 2.4 m, enough for a 2-man abreast team. With this width, the ramp would have reached a level of 145 m, the base of the capstones, where it would have terminated in a level apex platform, built in wood, to provide the working space needed for operations there.

Lift from Top of Ramp

Once the ramp is too short for the teams, an alternative means of raising the blocks, which at this level would have weighed less than 2.5 tonnes, would have been required. In principle, a ramp could have been built up the south west arris with a slope of 42°, hauled up by a team standing on the course under construction, or the one below, and hauling on a rope which passed over a horizontal post at waist height. For the uppermost course, and allowing for friction losses over the post, a team of 48 men would have been required to haul up a 2.5 t block. There would have been space for a team this size on the uppermost course. However, the risks involved in hauling such heavy blocks up a steep slope are great, as they would have slid backwards under their own weight on any slope steeper than 14°, and quickly accelerated. Any momentary lapse by one of the team, or the breakage of a rope, would have had serious consequences.

Big ropes were in common use in Egypt from at least Old Kingdom times onwards, as for example seen in Khufu's solar boat. Ropes found in caves at Mersa/Wadi Gawasis on the Red Sea coast are dated to the 12th Dynasty (Veldmeijer *et al.*, 2008: 23) though a slightly younger date (early New Kingdom) was not entirely ruled out. A comprehensive analysis (Veldmeijer *et al.*, 2008) records that that rope diameters were up to 38 mm and the numerous coils with lengths up to 700 m. The authors discuss the factors governing strength but note that it is not possible to test the strength of 4000-year old ropes and make no estimates of breaking strain.

A modern 32-mm diameter manila rope has a breaking strain of 7.9 tonnes, and modern Egyptian palm fibre ropes have a breaking strain of about 5.0 tonnes (Parry, 2004: 60). To reduce the risks of accidents, a safe working load of 2.5 tonnes has been adopted in the calculations, suggesting that a single rope would have been sufficient to hoist the pyramidion. For stability, there might have been four ropes attached to the sled for lifting purposes, and strength is unlikely to have been an issue.

Although the arris ramp could have been used to raise blocks to the apex platform, lack of space for the haul teams meant that it would not have been an option for the capstones, necessitating a further solution for them. The alternative was to use sheerlegs, as suggested elsewhere (Brichieri-Colombi, 2015: 13) straddling the arris, with two intermediate steps between the 3rd and 4th south west corner platforms for the blocks to rest on while they were being attached to the next set of sheerlegs (figure 6). The haul team would have stood on the course under construction. For each lift, the sizes of teams needed would have depended on the angle to the horizontal of the haul rope, which determines the friction losses over the horizontal poles, and the lever arm – the distance between this rope and the point of rotation of the sheerlegs (on the line joining their feet). This distance could have been increased as required by lengthening the sheerlegs, allowing the haul team to be decreased in size until it could fit on the course under construction.

There are several solutions that would have worked. The sheerlegs illustrated in figure 6 correspond closely with those used by the author in an experiment with a 400 kg block to verify the practicability of the idea (Brichieri-Colombi, In Preparation). Table 3 shows the calculation of the team sizes would have been needed for each lift. Although there would have been enough space for the 57 men needed for the lift from step 3 to step 4, there would not have been enough for the 91 men needed for the lift from step 4 to step 5. At this point, therefore, a second pair of sheerlegs would have been required, linked to the first one by a rope from apex to apex. The effect of this is to greatly increase the lever arm and so reduce the size of the team required to 44, which is smaller than the team re-

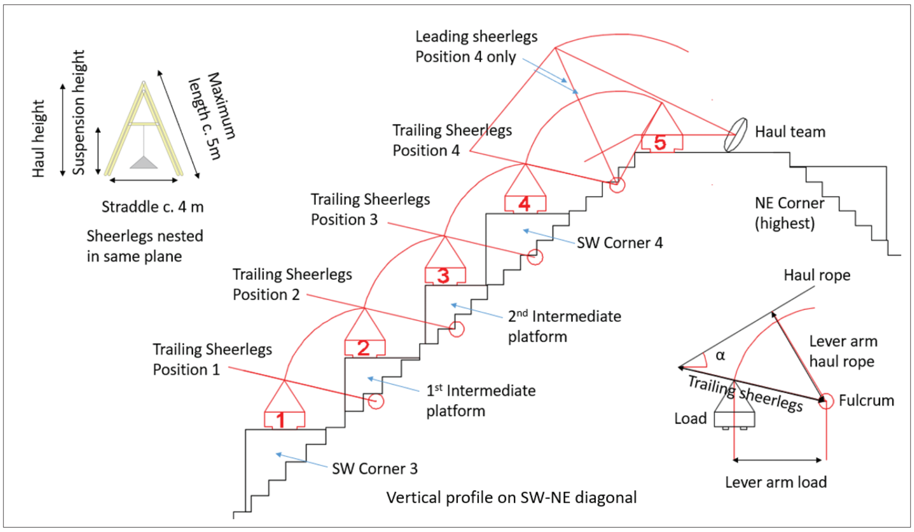


Figure 6. Raising the highest backing blocks.

	Max wt	Load moment	Sheerleg moment	Total moment	Lever arm	Bend angle	Bend loss	Team size
	t	t-m	t-m	t-m	m	°		no.
Lifting casing block to base of capstones								
1 to 2	2.46	7.64	0.05	7.68	3.47	30.9	13%	48
2 to 3					3.17	26.2	11%	51
3 to 4					2.71	19.8	8%	58
4 to 5					1.59	5.4	2%	93
4 to 5*					3.20	0.0	0%	45
Lifting pyramidion								
1 to 2	0.88	2.72	0.17	2.89	3.47	30.9	13%	18
2 to 3					3.17	26.2	11%	19
3 to 4					2.71	19.8	8%	22
4 to 5					1.59	5.4	2%	35
4 to 5*					3.20	0.0	0%	17
4 to 6*					3.04	0.0	0%	18
* with 2nd sheerleg								

Table 3. Team sizes for the final haul.

quired for an arris ramp. The same pair of linked sheerlegs could have been used to lift the pyramidion and other capstones directly from position 4 to their final position (figure 7). This would have required a team of 16 men, standing on the last two flights of the ramp (figure 8). Once the pyramidion had been placed, operations to trim the glacis slope and remove the ramps could be started.

Problems with Spiral Ramp Construction

Arnold (1991: 100) and others (*e.g.* Hawass & Brock, 2000: 245; Hitchens, 2010: 131-132; Isler, 2001: 215-216; Lehner, 1997: 216) identify several objections to the use of spiral ramps:

1. Haul teams could not have manoeuvred heavy blocks around the corners of the ramps.
2. Ramps would have prevented surveyors from maintaining building control by sighting up the arrises and apothems.
3. The ramps could not have been founded on the sloping face of the pyramid.

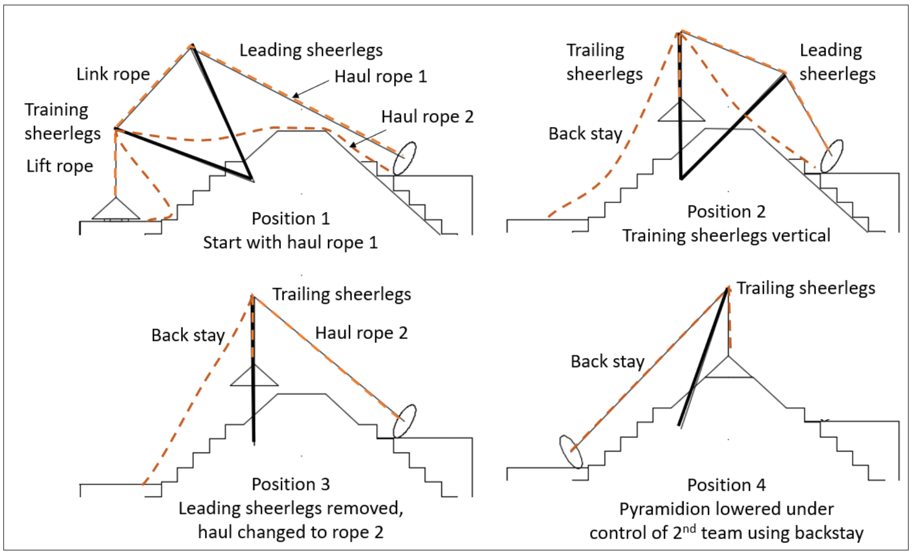


Figure 7. Raising the pyramidion.

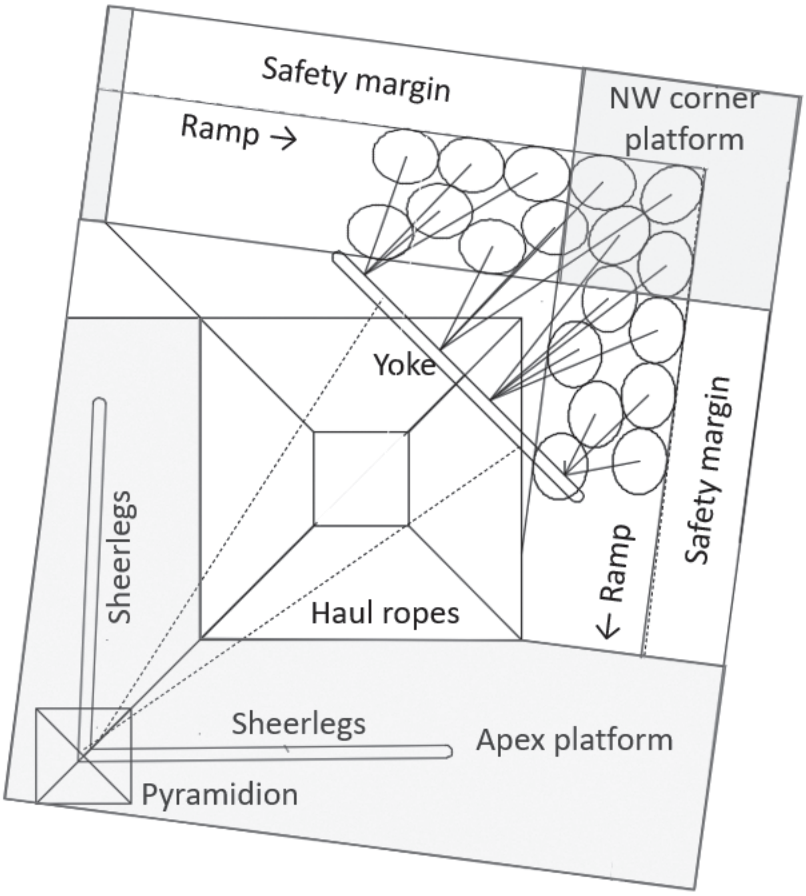


Figure 8. Team configuration for the final lift.

The first objection was addressed previously (Brichieri-Colombi, 2015: 11), who showed that the problem could have been solved using linked spars. The second objection could have been solved if, the ramps had vertical ends as well as vertical sides. Each flight of the ramp could have terminated with a vertical wall at the arris at each end, with a gap 2 m wide where it crossed the apothem. The resulting gaps would have been spanned with timber bridges or corner platforms (figure 9) with a 2 x 2 m viewport on the inside edge to allow the surveyors a clear view up the lines of the arrises and apothems. The weights of the sleds negotiating the corner platforms would not have exceeded 6 t, a load easily supported by the

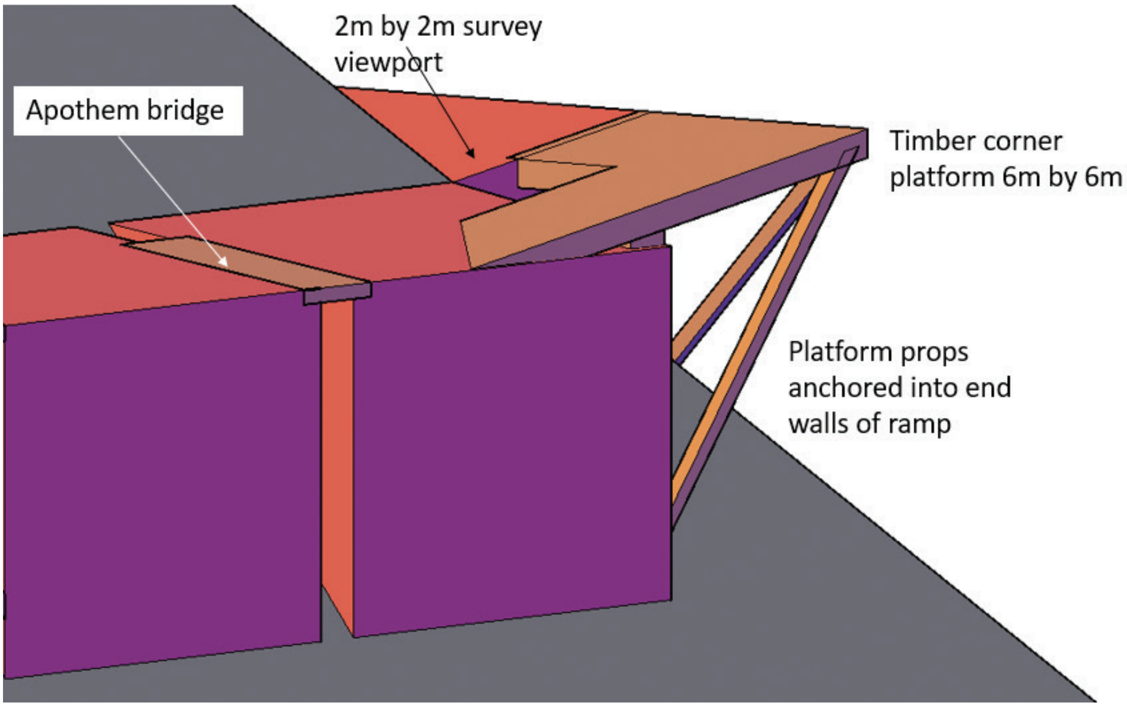


Figure 9. Corner platform.

timber framework shown in figure 9. The 75 t load on the first flight would have been on a sled at least 8 m long (the length of the granite beam) and the stresses on the timbers of a 2 m span bridge over the apothem would also have been low. The third objection is a major issue that concerns the design and construction of the ramp and is therefore examined in more detail in the next section.

Ramp Design and Construction

The pyramid consists of core materials including limestone rubble and blocks (ashlars) of various shapes and sizes many of them prismatic, sand, tufla clay and gypsum, which is surrounded by courses of backing blocks of which the great majority are roughly prismatic. The exact composition of the core is irrelevant to the ramp design, provided the largest blocks are lighter than the gable beams above the King's Chamber, or the backing blocks above the level of these beams. The courses of backing blocks occasionally contain two superimposed blocks, or oversize blocks that intrude into the next course (Arnold, 1991: 168). Observations of the blocks by the author, and assuming their widths inwards are the same as their average length parallel to the side and that half are laid as headers and the others as stretchers (none are laid diagonally), showed that we can estimate their volume as 71,000 m³. These were originally surrounded by 103,000 m³ of facing blocks with outer faces that projected beyond the glacis slope of the pyramid and were trimmed to their final slope once all of them had been placed, working down from the top (Lehner, 1997: 223). It seems likely that, at a minimum, the base of these blocks projected 5 cm beyond the glacis face to protect the bottom outer edge from damage in transit (Arnold, 1991: 171). We do not know the extent to which the glacis face was trimmed to leave a boss prior to placing, and diagrams and photos (Lehner, 1997: 220-221) suggest that it could have been very little, or almost all the stock. Importantly, Lehner (2003: 40) notes that "Hawass has excavated at the basis of Khufu's queens' pyramids to reveal that the builders here left a great deal of extra stock in rough steps protruding beyond the plane of the pyramid. These may be unusual because they are part of the foundation, but if this much extra stock was led on higher casing stones, it might well have supported a spiral ramp."

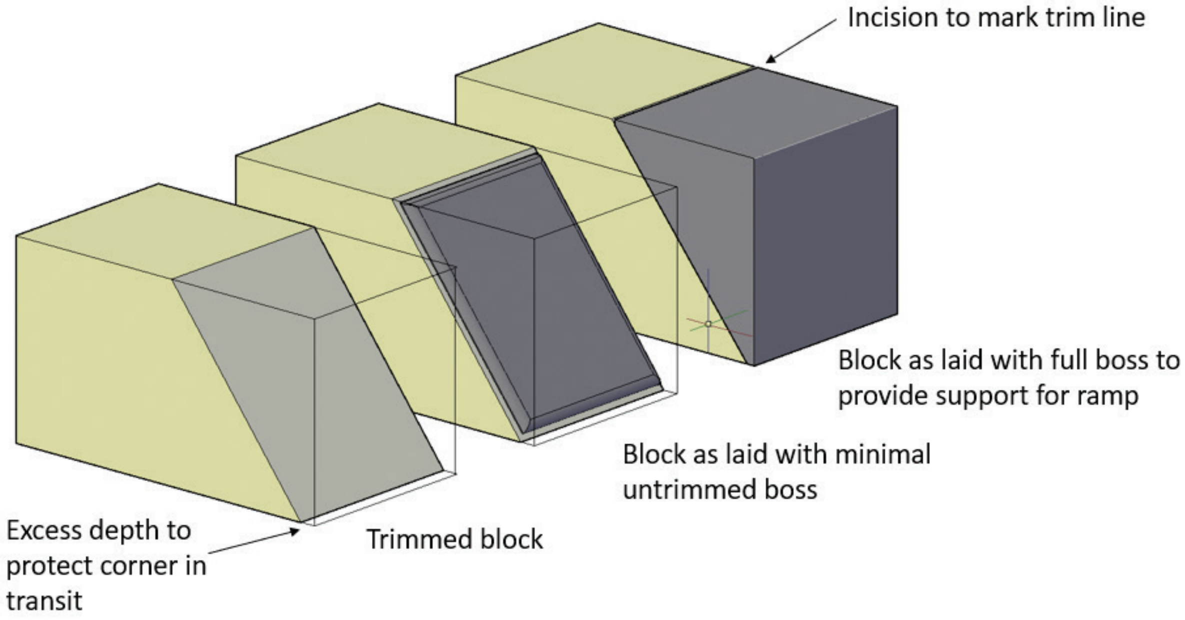


Figure 10. Casing block alternatives

There is little reason for extra stock to be left on the base course only, and this finding provides substantial evidence that enough stock was left on which to found a spiral ramp.

Incisions were made on the top surface parallel to the line of the face (Lehner, 1997: 221) to indicate to the masons where they should cease trimming (figure 10). It seems probable that these lines were incised in the top of each course immediately after the course was laid, so that even if the back faces were slightly out of alignment (or had been slightly displaced as a backing block was laid), the glacis face could be aligned precisely with the carefully positioned corner and apothem blocks. This process would have permitted the apothem block to be set slightly back from the lines joining the corners to create the hollowing of the faces noted by Petrie (1893: 43-44).

The facing blocks would have been roughly prismatic when quarried, so the total volume to be trimmed would have been the same, and in all cases, the back, sides, and possibly the top would have been trimmed off-site. For the front face, in the case of the minimal boss, some of the trimming would have been done off-site and the rest *in situ*. In the case of a full boss, all would have been trimmed *in situ*. For facing blocks of limestone, cut with copper chisels, there would have been little advantage in time and effort one way or the other. Trimming the granite blocks on Khafre's pyramid would have been much harder, making it more likely that the bulk of the work was done at quarries using pounders.

Had the facing blocks been laid with the glacis face untrimmed, the top surface would have provided a solid horizontal foundation for the spiral ramp and an easy way for workers to ascend and descend the pyramid face. It seems unlikely that the Egyptians would not have appreciated this advantage. It is also possible that untrimmed facing blocks were left only where required for the ramp.

The clearest example of the traces of a major ramp with vertical sides can still be seen at Meidum, as shown in the photo taken by Arnold (1991: 83). He records that the width of the trace varied from 5.36 m to 4.95 m on the 5th and 6th steps, i.e. between levels 43 m and 61 m. This corresponds to a batter on the sidewalls of the ramp of 88 vertical to 1 horizontal, a negligible difference from vertical, on a ramp up to 60 m high. Arnold (1991: 83) goes on to assert that "Since no ramp with nearly vertical walls could stand up to a height of 55 meters, inclined outer layers would have been added from the two sides", with no supporting evidence or indication of what would have been the minimum batter to ensure the ramp was stable.

The maximum unsupported height of the 12 m wide spur ramp would have been 47 m, the maximum height of the wall of the first flight of spiral ramp 15 m, and of further flights 7 m. By comparison, the 14th century Salvucci south tower in San Gimignano, Italy, built of 235 cm thick ashlar on a 7 m wide square base in an earthquake prone area, is 43 m high. There are many such medieval towers in Italy that have stood for centuries.

The most common kind of ramp that has been found in Egypt is one with lateral and transverse walls of stone forming interior cages containing rubble set in tafla (Arnold, 1991: 82-97; Hawass, 1998: 53) These would have been cheap and easy to build, and suitable for permanent ramps of moderate height. The design is less suitable for high ramps, where the cages would have acted as silos containing granular material which, despite the presence of tafla, would exert horizontal pressure on the walls. For a wall 20 m high, retaining material with a density of 2 t/m³ and with an angle of internal friction of 40°, this pressure is 240 t per metre run at ⅓ the height of the wall, and more if there had been a superimposed load. The pressure would be reduced by the cohesiveness of the tafla (clay), which would vary by season and the degree to which lubricants penetrated the mix. The batter would have to be shallower than 1:1 to prevent the wall either overturning or the courses of the wall sliding laterally outwards over one another. The lateral pressure would have been negligible if the ramp had been made entirely of ashlars similar to backing blocks and stabilized with gypsum mortar or tafla, as such blocks do not generate any horizontal forces when stacked on each other.

As noted above, the volume of ashlars in the backing and facing blocks would have been 182,000 m³, plus any ashlars in the core, while the volume of the spur ramp would have been 90,000 m³ and in the spiral ramp 20,000 m³ for a total of 110,000 m³. Thus, if the ramp had been constructed entirely of ashlar, it would have increased the total ashlar volume by 60%. This would not have been wasted work: once the ramps had been removed, these ashlars would have been reused later in the many tombs surrounding the pyramid. With a lower safety factor, the ramps could have been constructed with 2 m thick ashlar walls and consolidated rubble fill, which would have resulted in a saving of 60,000 m³ of extra ashlars. Whether the architect would have wanted to put the entire project at risk for such a minimal saving is questionable.

The process of building the spiral ramp would have been simple, as shown in figure 11. When a course had been finished, a course of ashlars tapering from 6 m wide to zero over a length equal to the ramp *seked* (6) multiplied by the course height would have been added to the outside of the facing blocks. All but the last section rests on the earlier courses of the ramp, and the last one on the facing blocks of the course below. If ashlars had been at a premium, the addition could have been made with only the exterior wall of ashlar, but the same weight of material would have had to be raised. The 60% increase in the number of workers required to shape quarried blocks into ashlars would have been a small fraction of the number needed to build an enveloping ramp of the kind proposed by Lehner (1985: 129-132).

Optimization of Slope

The work effort that was required to build the pyramid can be measured by the work that was done against gravity to raise the blocks, the men that haul them, and the work done against friction under the sled (both measured in millions of tonne-metres). The blocks are assumed to originate either from the main quarry

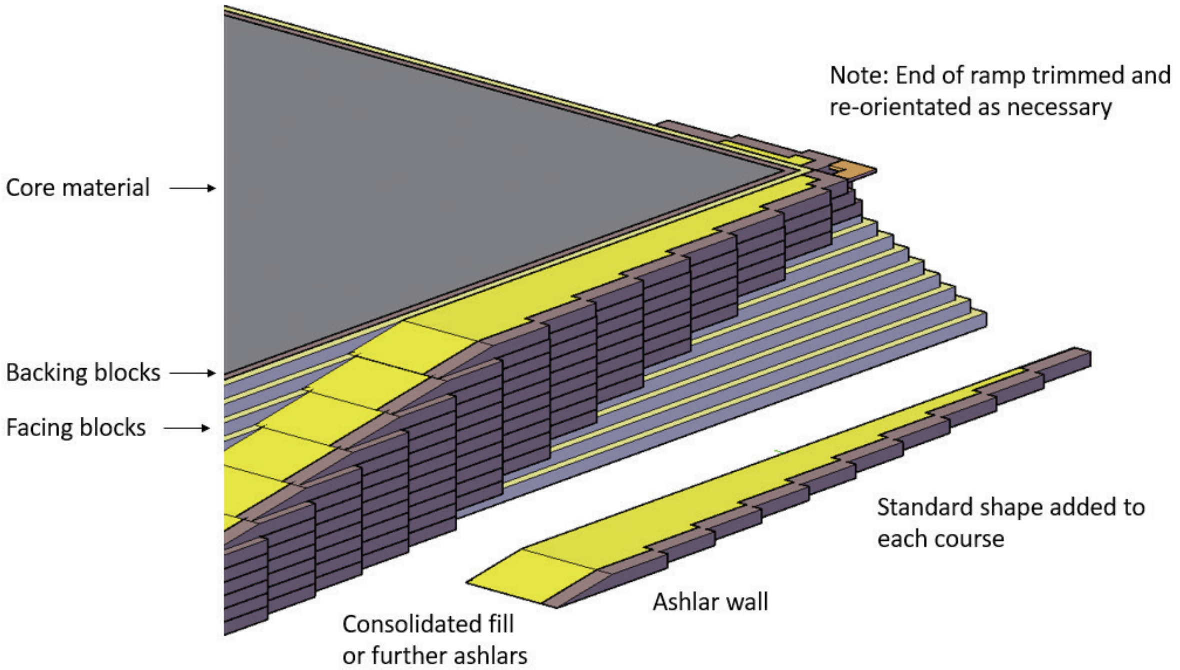


Figure 11. Spiral ramp construction.

or the harbour (for the blocks from Tura), from an average level 20 m below foundation level. The work done raising the blocks is estimated by summing the product of weight of each course (and associated section of ramp) by its level above the source. The work done raising the men is estimated at 87% of this number, corresponding to the weight of the haul team as a percentage of the load. Fonte (2007) and De Haan (2009) make similar calculations, but omit the work done by men raising their own weight.

The work against friction is estimated by summing the product for each course of the weight of the course, the force required per tonne on each haul surface, and the length of the surface. The surfaces considered are:

1. the quarry road up to the causeway due south of the pyramid axis.
2. the causeway from there to the start of the ramp.
3. the horizontal section of the ramp (for levels below that of the first south west corner).
4. the sloping section of the ramp.
5. the haul across the course under construction.

Length (a) is fixed at 385 m, which is the distance to the harbour. This is also a reasonable estimate of the average length of quarry roads feeding the causeway. Lengths (b), (c), and (d) vary with slope, while length (e) would have averaged 2.65 times the side length of the course under construction, assuming the distribution on the course was via a square ring road that enclosed an area equal to half that of the course (figure 12).

The total work done, and the volume of the ramp as a percentage of the pyramid volume, both diminish as the *seked* of the ramp increases (figure 13). However, there are other constraints that would have had to be respected and which would have limited the choice of slope:

1. The level of the main ramp at the first north west corner had to be above the level set by the teams hauling the beams above the King's Chamber. Both levels depend on the slope of the ramp.
2. The maximum team size for the heaviest beams would not have exceeded the gang size of 1000 men (Lehner, 1997: 225).

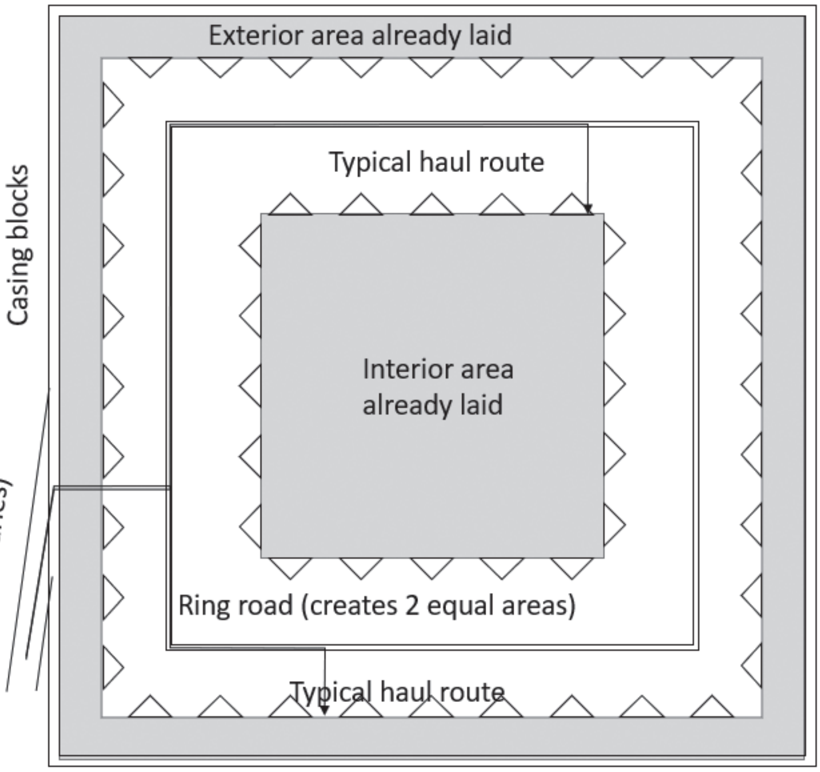


Figure 12. Distributing core blocks on course under construction

3. The residual lift for the pyramidion would have been limited to around 10 m.
4. The maximum unsupported height of the spur ramp, where it crosses over the south edge of the pyramid, is unlikely to have exceeded 54 m (the height of the Torre Grande in San Gimignano, Italy).

Over a range of slopes from 5 to 7 *seked*, only those between 5.5 to 6 *seked* met all four constraints. The work is lowest with 6-*seked* ramp (figure 13), so this would have been the optimal choice for the pyramid builders. The volume of the ramp is then less than 5% of the pyramid volume.

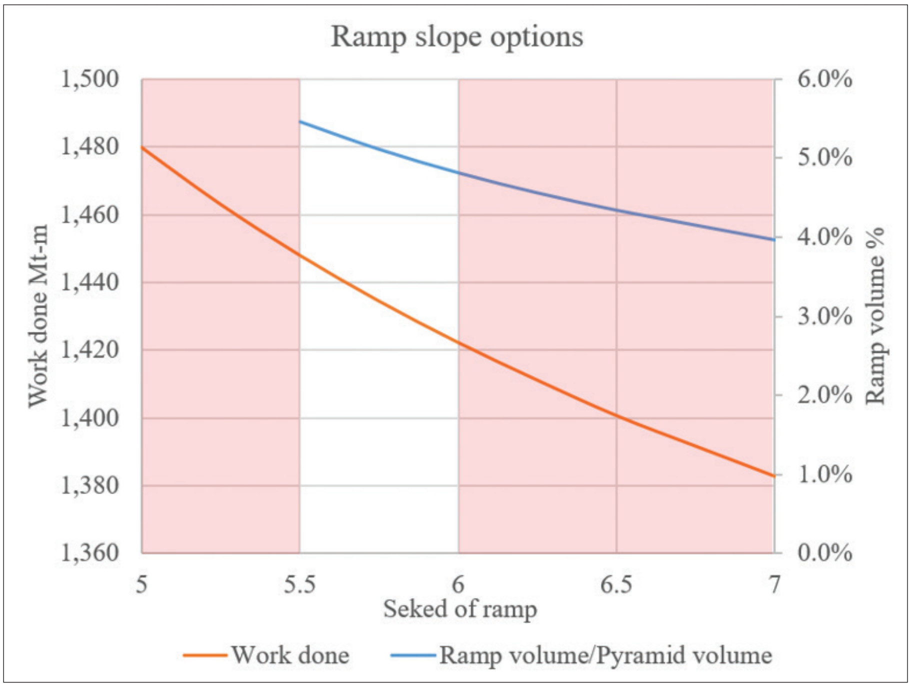


Figure 13. Variation of work done and ramp volumes with ramp seked.

With this slope, 61% of the work done would have been against friction, compared with 21% to raise the blocks and 18% to raise the haulers themselves. Varying the friction coefficient by ± 0.05 varies the work done by ±14% either way. This makes clear the importance of introducing methods to keep friction as low as possible.

Conclusion

Spiral ramps with a slope of 1 in 6 would have provided a simple, feasible and economic method for pyramid construction, but many authors have dismissed them because they pose several problems that would have had to be overcome. This paper demonstrates that none of these problems is insoluble, and that the Egyptians could have overcome them using simple methods that were available in Old Kingdom times. It is evident that the pyramid builders were constantly trying new and different approaches, and they had had 80 years of accumulated experience by the time they came to build the Great Pyramid in which to the perfect their ideas. There is no reason for us to believe they had some hereto undiscovered method for raising large blocks other than hauling them up on sledges and, for the final lift of a small number of blocks at the apex, with sheerlegs, unless an alternative can be shown to be more viable under specific criteria.

The paper endorses the approach by de Haan, which considers minimum energy use as a criterion for selecting among competing hypotheses. For this approach to work, there needs to be a consistent set of assumptions about the characteristics of the pyramid, the workforce, and any features. that impose specific constraints. It would then be possible to assess whether one hypothesis is more efficient than another due to the method adopted rather than the assumptions made. The first table in present paper is intended to be a starting point for such a set of assumptions about the data. The objective would then be to find the hypothesis which is consistent with all known facts and constraints, and requires the lowest energy input using the agreed data set. The data set may change over time as archaeologists and experts from other disciplines provide more facts, but it may well be that this does not revise the ranking of different hypotheses.

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