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N-Generated T-Intuitionistic Fuzzy Subgroups

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ABSTRACT

In this paper a notion of N- generated T- intuitionistic fuzzy set and N- generated T- intuitionistic fuzzy subgroup (normal subgroup) are defined and discussed. The homomorphic behaviour of N-generated T- intuitionistic fuzzy subgroup (normal subgroup) and their inverse homomorphic images has been obtained. Some related results have been derived.

1. INTRODUCTION

After the introduction of the concepts of fuzzy sets by zadeh several researches were conducted on the generalizations of the notion of fuzzy sets. The idea of "Intuitionistic fuzzy set" was first published by Atanassov, as a generalization of the notion of the fuzzy set we introduce the notion of N- generated T- intuitionistic fuzzy set and then defined N- generated T- intuitionistic fuzzy subgroup (normal subgroup) of a group and their properties. This paper discussed about the N-generated T- intuitionistic fuzzy subgroup and their corresponding propositions.

2. PRELIMINARIES

2.1 Definition

Let G be a group , a fuzzy subset A f G is said to be a fuzzy subgroup of G. If,

A(xy) ≥minA(x)A(y)

A(x^{-1}) ≥A(x) for all (x,y) ∈ G.

2.2 Definition

Let G be a group an N- generated fuzzy subset B= (ϕ_B, ψ_B) of G is called N-generated fuzzy subgroup of G if,

$\lambda(xy) \geq \min \lambda(x)\, \lambda(y)$

$\lambda(x^{-1}) \geq \lambda(x)$ for all x,y∈ G.

where, $\lambda(x) = \frac{1}{k}\sum_{i=1}^{k} \phi^{n}{}_{i}(x)$

λ (y) $= \frac{1}{k}\sum_{i=1}^{k} \phi^{n}{}_{i}(y)$

λ (xy) $= \frac{1}{k}\sum_{i=1}^{k} \phi^{n}{}_{i}(xy)$.

2.3 Definition

Let G be a group. An N- generated intuitionistic fuzzy subset (N-IFS) B= (ϕ_B, ψ_B) of G is called N- generated intuitionistic Fuzzy Subgroup(N-IFSG) of G if,

$\Phi(\mathrm{xy}) \geq \min\phi(\mathrm{x}), \phi(\mathrm{y})$

$\phi(\mathrm{x}^{-1}) = \phi(\mathrm{x})$ for all x,y$\in$ G

Where ϕ (x) $= \frac{1}{N}\sum_{i=1}^{N} \phi^{n}{}_{i}(x)$

ϕ (y) $= \frac{1}{N}\sum_{i=1}^{N} \phi^{n}{}_{i}(y)$

ϕ (xy) $= \frac{1}{N}\sum_{i=1}^{N} \phi^{n}{}_{i}(xy)$.

$\psi(\mathrm{xy}) \leq \min\psi(\mathrm{x}), \psi(\mathrm{y})$

$\psi(\mathrm{x}^{-1}) = \psi(\mathrm{x})$ for all x,y$\in$ G

where, ψ (x) $= \frac{1}{N}\sum_{i=1}^{N} \psi^{n}{}_{i}(x)$

ψ (y) $= \frac{1}{N}\sum_{i=1}^{N} \psi^{n}{}_{i}(y)$

ψ (xy) $= \frac{1}{N}\sum_{i=1}^{N} \psi^{n}{}_{i}(xy)$.

B$(x^{-1}) = B(x)$for all x,y$\in G$

B(xy)= $\frac{1}{k} \sum_{i=1}^{k} \phi^{n}{}_{i} (xy)$

2.4 N- Generated T- Intuitionistic fuzzy subgroup

Let G be a group. A N-generated T-intuitionistic fuzzy subset (N-T-IFS) B^{T}=(ϕ_{B^T}, ψ_{B^T}) of G is called N-generated T- intuitionistic fuzzy subgroup (N-T-IFSG) of G if,

$\phi_{B^T}(\mathrm{xy}) \geq \min\phi_{B^T}(\mathrm{x}), \phi_{B^T}(\mathrm{y})$

$\phi_{B^T}(\mathrm{x}^{-1}) = \phi_{B^T}(\mathrm{x})$ for all x,y$\in$ G

where,ϕ_{B^T} (x) $= \frac{1}{N}\sum_{i=1}^{N} {\phi_{B^T}}^{n}{}_{i}(x)$

ϕ_{B^T} (y) $= \frac{1}{N}\sum_{i=1}^{N} {\phi_{B^T}}^{n}{}_{i}(y)$

ϕ_{B^T} (xy) $= \frac{1}{N}\sum_{i=1}^{N} {\phi_{B^T}}^{n}{}_{i}(xy)$.

$\psi_{B^T}(\mathrm{xy}) \leq \max\psi_{B^T}(\mathrm{x}), \psi_{B^T}(\mathrm{y})$

$\psi_{B^T}(\mathrm{x}^{-1}) = \psi_{B^T}(\mathrm{x})$ for all x,y$\in$ G

where, ψ_{B^T} (x) $= \frac{1}{N}\sum_{i=1}^{N} {\psi_{B^T}}^{n}{}_{i}(x)$

ψ_{B^T} (y) $= \frac{1}{N}\sum_{i=1}^{N} {\psi_{B^T}}^{n}{}_{i}(y)$

ψ_{B^T} (xy) $= \frac{1}{N}\sum_{i=1}^{N} {\psi_{B^T}}^{n}{}_{i}(xy)$

2.5 Definition

Let G be a group. An N-Generated fuzzy subgroup B=$\{\phi_B, \psi_B\}$ of group G is said to be N-Generated intuitionistic fuzzy normal subgroup (N-IFSG) of G. if,

$\phi_B(y^{-1}xy) = \phi_B(x)$

$\psi_B(y^{-1}xy) = \psi_B(x)$ for all x,y$\in$G.

where,

ϕ_B $(y^{-1}xy)$ $= \frac{1}{N}\sum_{i=1}^{N} {\phi_{B_i}}^{n}(y^{-1}xy) = \frac{1}{N}\sum_{i=1}^{N} \phi_{B_i}(x)$

$$\psi_B\ (y^{-1}xy\ ) = \frac{1}{N}\sum_{i=1}^{N}\ \psi_{B_i}{}^{n}(y^{-1}xy) = \frac{1}{N}\sum_{i=1}^{N}\ \psi_{B_i}(x)$$

2.6 Definition

Let G be a group. An N-Generated intuitionistic fuzzy subgroup B=$\{\phi_{B^T},\psi_{B^T}\}$ of group G is said to be N-Generated T-Intuitionistic fuzzy normal subgroup (N-generated T-IFNSG) of G. if,

$\phi_{B^T}(y^{-1}xy) = \phi_{B^T}(x)$

$\psi_{B^T}(y^{-1}xy) = \psi_{B^T}(x)$ for all x,y$\in$G. where,

$$\phi_{B^T}\ (y^{-1}xy)\ = \frac{1}{N}\sum_{i=1}^{N}\ \phi_{B_i{}^{T_i}}(y^{-1}xy) = \frac{1}{N}\sum_{i=1}^{N}\ \phi_{B_i{}^{T_i}}(x)$$

$$\psi_{B^T}\ (y^{-1}xy\ ) = \frac{1}{N}\sum_{i=1}^{N}\ \psi_{B_i{}^{T_i}}(y^{-1}xy) = \frac{1}{N}\sum_{i=1}^{N}\ \psi_{B_i{}^{T_i}}(x).$$

2.7 N- Generated Intuitionistic Fuzzy Left Coset:

Let G be a group and B be N-Generated intuitionistic fuzzy subgroup (N-IFSG) of group G. Let x$\in$ G be a fixed element. Then for every element s$\in$ G. We define,

(xB) s ={ $\phi_{xB}(s)$, $\psi_{xB}(s)$} where,

$\phi_{xB}(s) = \phi_B(x^{-1}s)$ and

$\psi_{xB}(s) = \psi_B(x^{-1}s)$,where,

$$\phi_{xB}(s) = \frac{1}{N}\sum_{i=1}^{N}\phi^{n}{}_{i}(s)$$

$$\psi_{xB}(s) = \frac{1}{N}\sum_{i=1}^{N}\psi^{n}{}_{i}(s)$$

2.8 N- Generated T- Intuitionistic Fuzzy Left Coset

Let G be a group and B^T be N-Generated T- intuitionistic fuzzy subgroup (N-T-IFSG) of group G. Let x$\in$ G be a fixed element. Then for every element s$\in$ G. We define,

(xB^T) s ={ $\phi_{xB^T}\ (s)$, $\psi_{xB^T}\ (s)$} where,

$\phi_{xB^T}\ (s) = \phi_{B^T}\ (x^{-1}s)$ and

$\psi_{xB^T}\ (s) = \psi_{B^T}\ (x^{-1}s)$,where,

$\phi_{xB^T}\ (s) = \frac{1}{N}\sum_{i=1}^{N}\phi^{n}{}_{B^T{}_i}(s)$ and $\psi_{xB^T}\ (s) = \frac{1}{N}\sum_{i=1}^{N}\psi^{n}{}_{B^T{}_i}(s)$

2.9 N- Generated T- Intuitionistic Fuzzy right Coset

Let G be a group and B^T be N-Generated T- intuitionistic fuzzy subgroup (N-T-IFSG) of group G. Let x$\in$ G be a fixed element. Then for every element s$\in$ G. We define,

($B^T x$) s ={ $\phi_{B^T\mathrm{x}}\ (s)$, $\psi_{B^T\mathrm{x}}\ (s)$} where,

$\phi_{B^T\mathrm{x}}\ (s) = \phi_{B^T}\ (x^{-1}s)$ and $\psi_{B^T\mathrm{x}}\ (s) = \psi_{B^T}\ (x^{-1}s)$, where,

$$\phi_{B^T\mathrm{x}}\ (s) = \frac{1}{N}\sum_{i=1}^{N}\phi^{n}{}_{B^T{}_i}(s)$$

$$\psi_{B^T\mathrm{x}}\ (s) = \frac{1}{N}\sum_{i=1}^{N}\psi^{n}{}_{B^T{}_i}(s)$$

3. N- Generated T- Intuitionistic Fuzzy Subgroup

3.1 Theorem

If A is a N-generated intuitionistic fuzz subgroup of a group of a group G, then A is also N-generated T-IFSG of G.

Proof

Let x,y $\in$ G be any element of group G.

$\phi_{A^T}\ \ (xy) = \min\{\phi_A\ (xy), T\}$

where $\phi_{A^T}\ (xy) = \frac{1}{N}\sum_{i=1}^{N}\ \phi^n{}_{A_i{}^{T_i}}\ (xy)$

$\geq min\ [min\ \{\phi_A\ (x), \phi_A\ (y)\}, T\}$

$= \min\ [\min\ \{\phi_A\ (x), T\}, \min\ \{\phi_A\ (y), T\}$

$= \min\ \{\phi_{A^T}\ (x), \phi_{A^T}\ (y)\}$

thus, $\phi_{A^T}\ (xy) \geq \min\ \{\phi_{A^T}\ (x), \phi_{A^T}\ (y)\}$

$\Rightarrow \phi^n_{A_i{}^{T_i}}\ (xy) \geq \min\ \{\phi^n_{A_i{}^{T_i}}\ (x), \phi^n_{A_i{}^{T_i}}\ (y)\}$

similarly,

$\psi_{A^T}\ (xy) \leq \max\ \{\psi_{A^T}\ (x), \psi_{A^T}\ (y)\}$

$\Rightarrow \psi^n_{A_i{}^{T_i}}\ (xy) \geq \max\ \{\psi^n_{A_i{}^{T_i}}\ (x), \psi^n_{A_i{}^{T_i}}\ (y)\}$

Hence the theorem.

3.2 Preposition

Intersection of two N-Generated T-IFSG's of a group G is also N-Generated T-IFSG of G.

Proof

Let x,y $\epsilon\ G$ be any element of the group G.

then, $\phi_{(A\cap B)^T}\ (xy) = \min\ \{\phi_{(A\cap B)}\ (xy), T\}$

where, $\phi_{(A\cap B)^T}\ (xy) = \frac{1}{N}\sum_{i=1}^{N}\ \phi^n{}_{(A\cap B)_i{}^{T_i}}\ (xy)$

$\geq min\ [\min\{\phi_A\ (xy), \phi_B\ (xy)\}, T]$

$= \min\ [\min\ \{\phi_A\ (xy), T\}, min\{\phi_B\ (xy), T\}]$

$= \min\ [\min\ \{\phi_{A^T}\ (xy), \phi_{B^T}\ (xy)\ \}]$

$= min\ [mi\,n\{\phi_{A^T}\ (x), \phi_{A^T}\ (y)\}, mi\,n\{\phi_{B^T}\ (x), \phi_{B^T}\ (y)\}]$

$= min\ [mi\,n\{\phi_{A^T}\ (x), \phi_{B^T}\ (x)\}]\ [min\ \{\phi_{A^T}\ (y), \phi_{B^T}(y)\ \}]$

$= \min\ \{\phi_{(A^T\cap B^T)}\ (x), \phi_{(A^T\cap B^T)}\ (y)\}$

$= \min\ \{\phi_{(A\cap B)^T}\ (x), \phi_{(A\cap B)^T}\ (y)\}$

thus, $\phi_{(A\cap B)^T}\ (xy) \geq\ \min\ \{\phi_{(A\cap B)^T}\ (x), \phi_{(A\cap B)^T}\ (y)\}$

$\Rightarrow \phi^n{}_{(A\cap B)_i{}^{T_i}}\ (xy) \geq\ \min\ \{\phi^n{}_{(A\cap B)_i{}^{T_i}}\ (x), \phi^n{}_{(A\cap B)_i{}^{T_i}}\ (y)\ \}$

similarly, we can show that,

$\psi_{(A\cap B)^T}\ (xy) \geq\ \max\ \{\psi_{(A\cap B)^T}\ (x), \psi_{(A\cap B)^T}\ (y)\}$

$\Rightarrow \psi^n{}_{(A\cap B)_i{}^{T_i}}\ (xy) \geq\ \max\ \{\psi^n{}_{(A\cap B)_i{}^{T_i}}\ (x), \psi^n{}_{(A\cap B)_i{}^{T_i}}\ (y)\}$

Also,

$\phi_{(A\cap B)^T}\ (x^{-1}) \geq\ \min\ \{\phi_{(A\cap B)^T}\ (x^{-1}), T\}$

$\geq min\ [min\ \{\phi_A\ (x^{-1}), \phi_B\ (x^{-1}), T\}$

$= \min\ [\min\ \{\phi_A\ (x^{-1}), T\}, \min\ \{\phi_B\ (x^{-1}), T\}$

$= \min\ \{\phi_{A^T}\ (x^{-1}), \phi_{B^T}\ (x^{-1})\}$

$= \min\ \{\phi_{A^T}(x), \phi_{B^T}\ (x)\}$

$= \phi_{A^T\cap\,B^T}\ (x)$

$= \phi_{(A\cap B)^T}\ (x)$

similarly, we can show that

$\psi_{(A\cap B)^T}\ (x^{-1}) = \psi_{(A\cap B)^T}\ (x)$

Hence A∩B is N-Generated T-IFSG of G.

3.3 Theorem

If A is IFNSG of a group G, then A is also T-IFNSG of G.

Proof

Let A be IFNSG of G then for any sϵG , we have sA = As
that is,

$$\phi_A (s^{-1}g) = \phi_A(gs^{-1})$$
$$\psi_A (s^{-1}g) = \psi_A(gs^{-1}) \text{ for all g}\epsilon\text{G.}$$

so,
min$\{\phi_A (s^{-1}g), T\}$ = min $\{\phi_A(gs^{-1}), T\}$ and
max$\{\psi_A (s^{-1}g), T\}$ = max $\{\psi_A(gs^{-1}), T\}$
that is,

$$\phi_{sA^T} (s^{-1}g) = \phi_{A^T s}(gs^{-1}) \text{ and}$$
$$\psi_{sA^T} (s^{-1}g) = \psi_{A^T s}(gs^{-1}) \text{ for all g}\epsilon\text{G.}$$

Thus, sA^T = A^Ts .
Hence the theorem.

3.4 Preposition

Let A be N-Generated T-IFNSG of a group G.
then, $\phi_{A^T}(xy) = \phi_{A^T}(yx)$ and
$\psi_{A^T}(xy) = \psi_{A^T}(yx)$ holds for all x,yϵG.
where, $\phi_{A^T} (xy) = \frac{1}{N}\sum_{i=1}^{N} \phi^n{}_{A_i T_i} (xy)$ and $\psi_{A^T} (xy) = \frac{1}{N}\sum_{i=1}^{N} \psi^n{}_{A_i T_i} (xy)$.

Proof

Let A be N-Generated T-IFNSG of a group G. Therefore, sA^T = A^Ts for all ,s ϵG.

$$\Rightarrow sA^T(y^{-1}) = A^T s(y^{-1}) \text{ for all } y^{-1}\epsilon\text{G.}$$
$$\Rightarrow \min(\phi_A(s^{-1}y^{-1}), T) = min(\phi_A(y^{-1}s^{-1}), T)$$
$$\Rightarrow \left(\phi_{A^T}(s^{-1}y^{-1})\right) = \left(\phi_{A^T}(y^{-1}s^{-1})\right)$$
$$\Rightarrow \phi_{A^T}\big((ys^{-1})\big) = \phi_{A^T}((sy^{-1}))$$
$$\Rightarrow \phi_{A^T}(ys)^{-1} = \phi_{A^T}(sy)^{-1}$$

as A is T-IFSG of G,
so, $\phi_{A^T}(g^{-1}) = \phi_{A^T}(g)$ for all g ϵG.
Since A be N-Generated T-IFNSG of a group G.
Therefore, $A^T s = sA^T$ for all s ϵG.

$$\Rightarrow sA^T(y^{-1}) = A^T s(y^{-1}) \quad \text{for all } y^{-1} \epsilon\text{G.}$$
$$\Rightarrow \max \{\psi_A(y^{-1}s^{-1}), T\}$$
$$\Rightarrow \max \{\psi_A(s^{-1}y^{-1}), T\}$$
$$\Rightarrow \psi_{A^T}(y^{-1}s^{-1}) = \psi_{A^T}(s^{-1}y^{-1})$$
$$\Rightarrow \psi_{A^T}(ys^{-1}) = \psi_{A^T}(sy^{-1}) \text{ remove the inverse}$$
$$\Rightarrow \psi_{A^T}(ys) = \psi_{A^T}(sy)$$

as A is N-Generated T-IFSG of G so,

$$\psi_{A^T}(g^{-1}) = \psi_{A^T}(g) \text{ for all g } \epsilon\text{G.}$$

where,

$$\phi_{A^T} (xy) = \frac{1}{k}\sum_{i=1}^{k} \phi^n{}_{A_i T_i}(x)$$
$$\psi_{A^T} (xy) = \frac{1}{k}\sum_{i=1}^{k} \psi^n{}_{A_i T_i}(x).$$

Hence the theorem.

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